Deconfinement in QCD at Finite Temperature

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We present a simple and intuitive picture for the deconfinement of quarks and gluons at finite temperature: as the temperature increases, QCD behaves like QED at $T = 0$. We show this by calculating the QCD coupling constant as a function of the temperature and of the external momenta used to probe quarks and gluons.

The study of QCD at finite temperature and density is of crucial importance for the understanding of hadron formation in the early universe: as the temperature decreased, quarks and gluons must have experimented a phase transition from a quark-gluon dominated universe to a hadron dominated universe. Efforts to recreate the quark ↔ hadron transition are currently being fully implemented at RHIC, where it is hoped that clear signals of a quark-gluon plasma phase will be detected.

It is also currently believed, thanks to lattice simulations of finite temperature QCD [1], that a deconfined phase is expected to exist at temperatures above (and around) $0.2\ GeV$. However, we still do not have a clear picture of how QCD itself changes from a confining to a non-confining theory. That is, the quarks which were originally enslaved inside a region of $\sim 1\ fm$ are now allowed to propagate over a larger portion of space, outside the region occupied by the original nucleon. This is the strict sense that the word deconfinement is used througout this work. On the other hand, the difference between a non-confining theory, QED in the case, and a confining theory, QCD, resides exactly in the fact that the later has a gauge self coupling[1]. Physically, it may help to think that the deconfinement proccess considered here involves a transition from a non Abelian to an Abelian theory. In this sense, we think that in a deconfined phase, QCD should be "less" non Abelian than in a confined phase.

It is well known that at $T = 0$, besides a color factor, the QED and QCD β functions would be the same if only fermion loops existed. The difference in sign between the QED and QCD β functions comes, essentially, from the gluon and ghost loops in the gluon self energy (plus, of course, the vertex corrections). Alternatively, we notice that the two beta functions are the same in the limit that the number of colors, N_c, goes to zero. At finite temperature, it would be natural to expect that the same kind of limit exists. We will see in this letter that, in fact, QCD at high temperature and small external momenta squared behaves like QED at $T = 0$, a conclusion naturally drawn from the QCD β function at finite temperature. However, the calculation of this quantity has been plagued with dificulties, most of them related to the lack of gauge invariance [3–5] and the dependence of the renormalized coupling at finite temperature on the vertex chosen to renormalize it [6–8].

To ensure the gauge invariance of the β function means to ensure the gauge invariance of the vertex and self energy functions. Our freedom to choose among the different vertices (quark-gluon, gluon-gluon, ghost-gluon) should not affect the calculation of the running coupling. In general, we have:

$$\alpha_s = Z_\alpha^{-1}\alpha_B, \tag{1}$$

where Z_α is the renormalization constant of the coupling. If the gluon-ghost vertex is used for the renormalization, then:

$$Z_\alpha = \frac{\tilde{Z}_{T1}}{\tilde{Z}_{T3}Z_{T3YM}}, \tag{2}$$

with $\tilde{Z}_{T1}$, $\tilde{Z}_{T3}$, and Z_{T3YM}, the gluon-ghost vertex, the ghost propagator, and the gluon propagator renormalization constants, respectively, calculated at finite temperature. Alternatively, we could use the triple gluon vertex:

$$Z_\alpha = \frac{Z_{T1YM}}{Z_{T3YM}^2}. \tag{3}$$

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[1] QED at strong coupling can also have a confining phase [2]. However, in our case, we will be considering only the region of small external momentum where, as usual, $\alpha_s^{QED} \sim 1/137$.

Here, Z_{T1YM} is the renormalization constant of the triple gluon vertex. A similar expression exists for the quark vertex.

In a consistent calculation, Z_α is independent of the vertex used - at zero temperature this is a well known consequence of the Slavnov-Taylor identities. At finite temperature the situation is the same, as long as we restrain ourselves to the dominant, gauge invariant, terms as given by the Hard Thermal Loop (HTL) resummation program of Braeten and Pisarsky [9]. In fact, within this program one can show that the dominant terms obey the Abelian Ward identities [9,10]:

$$k^\mu \Pi_{\mu\nu}(k) = 0, \tag{4}$$

$$k^\mu \Gamma_{\mu\nu\rho}(k, p, q) = \Pi_{\nu\rho}(q) - \Pi_{\nu\rho}(p). \tag{5}$$

The HTL approximation consists of disregarding the *external* momenta in the numerators of the loop integrals, as the main contribution to these integrals comes from the region where the *internal* momenta are of order of T, with T taken to be large. Hence, if we compare the gluon, quark and ghost self energies, we see that the later is subleading to the formers because it does not have enough powers of internal momenta in the numerator to produce the leading (T^2) behavior in the temperature. From Eq. (5) we see that the ghost vertex will be subleading with respect to the quark and gluon vertices as well. Taking this into account, it follows from Eqs. (2) and (3) that the thermal parts of the triple gluon vertex renormalization constant and of the gluon self energy renormalizion constant should be equal:

$$Z_{T1YM}^{(leading\ T^2)} = Z_{T3YM}^{(leading\ T^2)}. \tag{6}$$

This relation is valid only for the dominant, gauge invariant, contribution. Z_α could also be calculated through the quark vertex. In this case there would appear an other constraint for the leading T^2 terms. Instead of Eq. (6), we would have that the renormalization constants for the quark vertex and the quark self energy should be the same.

A direct computation of the transverse part of the gluon self energy gives [11,12]:

$$\Pi_T^{(1)} = -\left(N_c + \frac{n_f}{2}\right)\frac{g^2 T^2}{12|\vec{k}|^2}\left[\frac{k_0}{|\vec{k}|}ln\left(\frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|}\right) - 2\frac{k_0^2}{k^2}\right], \tag{7}$$

where k is the four momentum of the external gluon. Only the dominant term in the high temperature expansion was written because of its gauge invariance. One can see that Eq. (7) vanishs for $k_0 << |\vec{k}|$, and that it is reduced to

$$\Pi_T^{(1)} = \frac{\alpha_s}{\pi}\left(N_c + \frac{n_f}{2}\right)\frac{2\pi^2}{3}\frac{T^2}{\mu_T^2} \tag{8}$$

for $k_0 >> |\vec{k}|$, where $\alpha_s \equiv g^2/4\pi$ and μ_T is a mass scale ($\mu_T^2 \equiv |\vec{k}|k_0$). Including the $T = 0$ part, we can calculate the renormalization constant for Π_T, which will depend on T/μ:

$$\Pi_T^R = Z_{T3YM}\Pi_T, \tag{9}$$

$$Z_{T3YM} = 1 + Z_{T3YM}^{(1)}$$
$$= 1 - \frac{\alpha_s}{\pi}\left(N_c + \frac{n_f}{2}\right)\frac{2\pi^2}{3}\frac{T^2}{\mu^2} - \frac{\alpha_s}{\pi}\left[N_c\left(\frac{13}{24} - \frac{a}{8}\right) - \frac{1}{6}n_f\right]\left(\frac{1}{\epsilon} + ln\mu^2\right), \tag{10}$$

where μ is the renormalization scale, a is the gauge parameter, and ϵ comes from the dimensional regularization of the $T = 0$ part of the gluon self energy [13]. The a dependence in Eq. (10) is cancelled by the remaining renormalization constants in Eqs. (2) and (3). Finally, we notice that the calculation of the triple gluon vertex gives a T^2/μ_T^2 dependence for the dominant term in the same form as Eq. (8), in a way that we can define a thermal renormalization constant which always satisfies Eq. (6).

The calculation of the thermal β function (including the $T = 0$ and the $T \neq 0$ parts), for a fixed temperature but an arbitrary renormalization point is now straightforward. Using Eqs. (1), (2), and (10), we have:

$$\mu\frac{d\alpha_s}{d\mu} = \frac{\alpha_s^2}{\pi}\left[-\frac{11}{6}N_c + \frac{2}{6}n_f - \mu\frac{dB(T^2, \mu^2)}{d\mu}\left(N_c + \frac{n_f}{2}\right)\right], \tag{11}$$

to order α_s^2, where $B(T^2, \mu^2) \equiv 2/3 \, \pi^2 \, T^2/\mu^2$. This is the only Renormalization Group Equation for α_s because there is only one renormalization scale for both the $T = 0$ and $T \neq 0$ parts of the renormalized gluon self energy. The solution of Eq. (11) is:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{\alpha_s(Q_0^2)}{4\pi} \left[\left(\frac{11}{3} N_c - \frac{2}{3} n_f \right) ln \left(\frac{Q^2}{Q_0^2} \right) + 4(B(T^2, Q^2) - B(T^2, Q_0^2)) \left(N_c + \frac{n_f}{2} \right) \right]}. \tag{12}$$

It is helpful to rewrite Eq. (12) in the same format of the $T = 0$ theory. To this end, we define an effective number of colors and an effective number of flavors:

$$N_c^{eff} = \left[1 + \frac{12}{11} \frac{B(T^2, Q^2) - B(T^2, Q_0^2)}{ln \left(\frac{Q^2}{Q_0^2} \right)} \right] N_c, \tag{13}$$

$$n_f^{eff} = \left[1 - 3 \frac{B(T^2, Q^2) - B(T^2, Q_0^2)}{ln \left(\frac{Q^2}{Q_0^2} \right)} \right] n_f. \tag{14}$$

With the help of Eqs. (13) and (14), the expression for the running coupling is written as:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{\alpha_s(Q_0^2)}{4\pi} \left[\frac{11}{3} N_c^{eff} - \frac{2}{3} n_f^{eff} \right] ln \left(\frac{Q^2}{Q_0^2} \right)}. \tag{15}$$

The interesting point of this set of equations is that for fixed Q_0^2 and T^2, $N_c^{eff} < N_c$ and $n_f^{eff} > n_f$ as $Q^2 \to 0$. It implies that at some small value of the external momenta squared, the coupling between quarks and gluons becomes that of an Abelian theory. To quantify this assertion, we make an explicit calculation of the coupling as a function of Q^2 for some fixed values of T. For $\alpha_s(Q_0^2)$, we use the experimental value measured at $m_Z \approx 91 \, GeV$ and at zero temperature [14]. That is, we assume that at such high values of the virtuality of the probe, temperatures of the order of $1 \, GeV$ are not relevant, an approximation which amounts to set $B(T^2 = 1 \, GeV^2, Q_0^2 \approx 83 \times 10^3 \, GeV^2) \approx 0$.

In Fig. 1 we show the behaviour of α_s for 3 values of the temperature. At $T = 0 \, GeV$ we have, as usual, that the coupling grows rapidly for $Q^2 < 1 \, GeV^2$. However, at $T = 0.5$ and $1 \, GeV$, $\alpha_s(Q^2)$ starts to change its behaviour in the region around $Q^2 = 10 - 20 \, GeV^2$. Instead of the rapid growth observed at the $T = 0$ case, for finite T there is first an almost Q^2 independence of the coupling, and then it *decreses* with Q^2. This behaviour of $\alpha_s(Q^2)$, for finite T and small Q^2, is that typical of an Abelian theory. In fact, if we define an effective β funtion at one loop,

$$\beta^{eff} = \frac{11}{3} N_c^{eff} - \frac{2}{3} n_f^{eff}, \tag{16}$$

we notice that it changes sign at some small value of Q^2. Figure 2 tells us that the effective β function changes sign around 0.05 and 0.2 GeV^2 for $T = 0.5$ and $1 \, GeV$, respectively. It means that at those values, the theory changes from a non Abelian one ($\beta^{eff} > 0$) to an Abelian one ($\beta^{eff} < 0$), implying that at high temperature quarks and gluons behave like electrons and photons.

Because the coupling does not grow at small Q^2 and high T, we are allowed to use perturbation theory in this region. A direct consequence of this fact is that we can, in principle, fix $\alpha_s(Q^2 \to 0, high \, T)$ using elastic scattering of a quark-gluon plasma by an electron beam.

The results presented here are easily extended to the case of a finite chemical potencial, μ_{cp}. One just has to replace T^2 by $T^2 + 3\mu_{cp}^2/\pi^2$ in the quark loops in the gluon self energy. The qualitative behaviour of α_s and β^{eff} are unchanged by the introduction of a chemical potential: the quark and gluon deconfinement at finite T and μ_{cp} still proceeds through a transition from a non Abelian to an Abelian theory.

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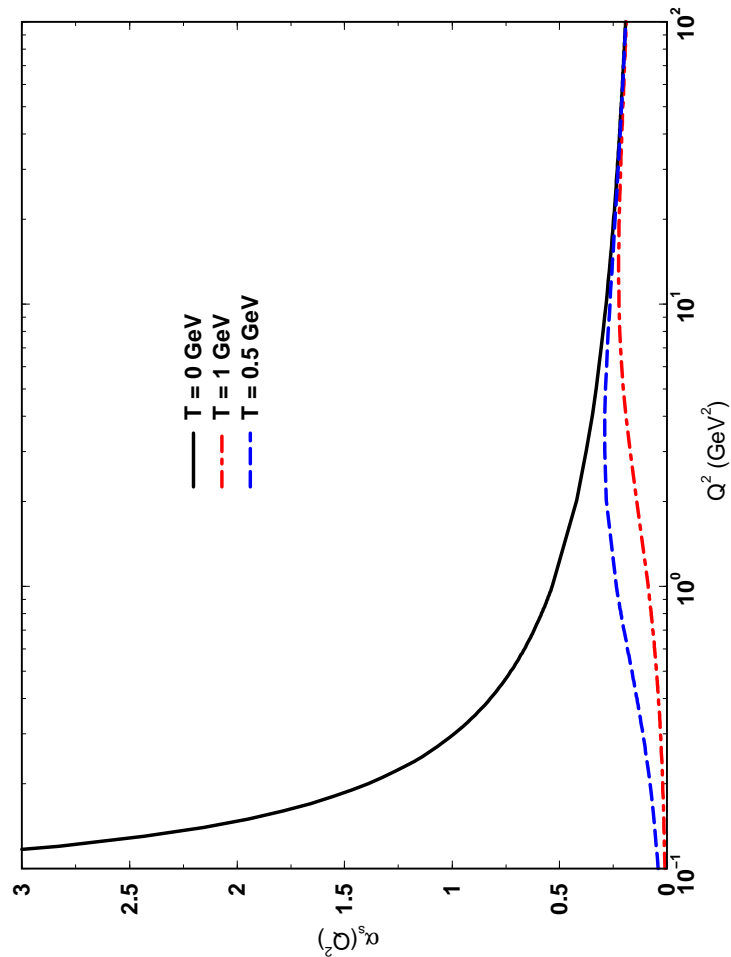
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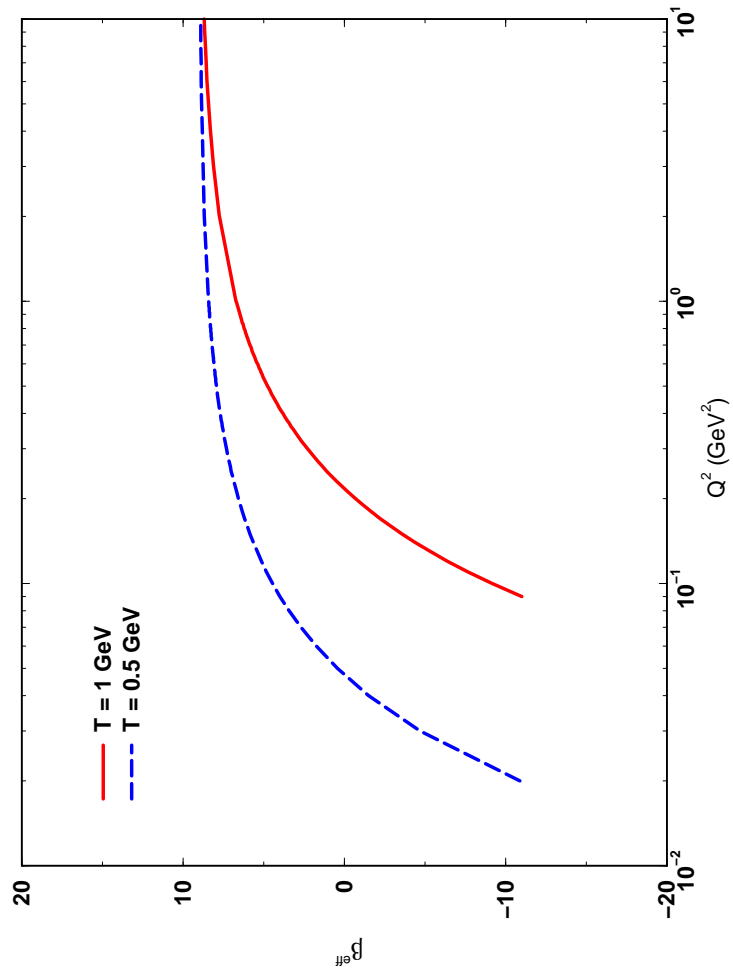FIG. 1. The strong coupling constant as a function of Q^2 calculated for 3 differerent values of the temperature.

FIG. 2. The effective beta function as a function of Q^2 for two values of the temperature. For large Q^2, β^{eff} tends to 9, which is the value of the β function at $T = 0$ for $N_c = n_f = 0$.